Oil price cycle and sensitivity model

Problem presented by

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Executive Summary

EPRasheed wishes to be able to model and predict oil prices out to a time-horizon of 2050, taking into account a number of known factors. These include the finite supply of oil, growing and shifting demand, the viability of alternative energy sources (at different pricing levels) and the interactions of oil producers and oil consumers, as they respond to current pricing levels.

The study group concluded that while 'prediction' of price in any meaningful sense was not viable, a model for scenario analysis could be realised. The model did not incorporate all of the factors of interest, but did model important time lags in the response of market players' future behaviour to current oil prices.

Consideration of the optimisation of supply through new capacity in the telecoms industry led to a generalisation of the standard Cournot-Nash equilibrium. This indicates how an output-constrained competitive market might operate. It enables identification of different pricing regimes determined by the level of competition and the resource limitations of particular supplier firms.

Two models were developed sufficiently to enable simulation of various conditions and events. The first modelled oil price as a mean reverting Brownian motion process. Strategies and scenarios were included in the model and realistic simulations were produced. The second approach used stability analysis of an appropriate time-delayed differential equation. This enabled the identification of unstable conditions and the realisation of price oscillations which depended on the demand scenarios.

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Contents

1 Introduction

1.1 Problem statement

(1.1.1) The problem is to develop a new model that attempts to model oil and gas prices over the next 30 or 40 years. Data are provided for the finding and lifting costs in different locations. Also provided are the historic and adjusted oil prices, back to 1861. The oil price cycle is sensitive to geopolitical, nonlinear factors as well as delayed time scales. The model should be able to capture these factors in some way.

(1.1.2) The model should consider this oil and gas price cycle sensitivity across different application areas, for example: transportation (road and air); power generation; heating; petrochemicals; surfacings (road, roof, cement) and lubricants. The model should also account for the discontinuities that occur when renewables/alternatives start becoming viable.

2 Practical and political considerations

2.1 Trends in oil consumption

(2.1.1) At the present time, oil consumption is accounted for by a range of application areas. Transportation in general is 50% of total, split between aviation (18%), cars (61%), trucks (21%). Other sectors include: Power Generation, 20%; Petrochemicals 10%; Heating 10%; Surfacing 5% and Lubricants 5%.

(2.1.2) In the future, it is expected that transportation will account for 75% of total, so becoming the primary source of oil consumption.

2.2 Oil refineries and oil refining

(2.2.1) **Time lags:** Two to five years is a realistic timescale to factor in when we consider the time it takes to get oil on the market. 'Easy' oil has already been accessed. Exploration for oil can take up to 3 years. If the oil is in a location where drilling is easy (such as the Saudi desert) it will take one year to drill the well. After this, the oil has to be piped. The infrastructure will have to be built and/or upgraded. It may take a further year or two to get the oil to market.

(2.2.2) **Light vs. heavy crude:** The world's current refining capacity is primarily geared towards refining light crude oil. This is a problem because most new crude coming on the market has a higher molecular weight and is therefore heavier and more viscous. Fractional distillation is the basic process of refinement. If there is more heavy oil, what that means is that there is less light oil (for transportation). Heavy oil must be further refined, resulting in extra cost and additional infrastructure requirements.

(2.2.3) **Sweet vs. sour crude:** Furthermore, crude oil from different countries has different molecular weight and different impurities, such as varying levels of sulphur. Sour crude is that which has more than 5% sulphur and oil with less than that percentage of sulphur is referred to as sweet crude. Most of the world's refining capacity is geared towards sweet crude. This is a problem because more and more sour crude is coming on to the market and this is difficult to refine, as there is not the refining capacity.

(2.2.4) A new installation for refining crude has a lifespan of about 20-25 years.

(2.2.5) Oil production that can be accounted for is 82M barrels per annum. However, the known consumption is 85M. The shortfall of 3M is accounted for by considering a variety of mechanisms, such as refinery gain[1] (typically a factor of 1.02); undeclared production[2]; there is a stock tank holding an unknown inventory[3].

2.3 The figure of 'known reserves'

(2.3.1) The 'known reserves' figure of 1.2 trillion barrels of oil is a moving target. These are the known reserves today for which there is infrastructure – i.e. we can produce them. However, it is known that there are several factors which will increase this number significantly: at a certain threshold price for crude, new technologies such as Enhanced Oil Recovery (EOR) become viable. Furthermore, there are areas of the world that are unexplored and there are areas that have been explored, but not yet exploited (i.e. known reserves for which there is no infrastructure currently in place).

(2.3.2) A company will quote its oil reserves, meaning known oil reserves. However, there will also be a *replacement factor* which is a multiplier that tells how these reserves will change over time. Most oil companies are (currently) replacing by a factor of 1.5 or 1.6. What this means is that each known barrel of oil in reserve today will be replaced with 1.5 or 1.6 barrels at some future date. This factor is dependent on oil price and fluctuates from year to year. It is generally an unknown number.

(2.3.3) Major multinational oil companies have a lot of cash. They have recently been redistributing their cash to shareholders, or using it to buy back their own shares. Note that they are not currently spending this cash heavily on research into oil exploration because of the low price of crude oil.

[1] The oil has a lower density in the petrol tank than in the ground.

[2] Declaring production brings with it obligations to pay taxes, etc.

[3] The US stock tank is only 200k barrels.

(2.3.4) State owned oil companies are used to balance the budget of the state to which they belong. The main source of cash for OPEC member states is from oil. When oil price drops below a certain point it is very damaging.

2.4 Alternative sources

(2.4.1) Alternative sources to oil could in future have a major impact on the oil price. Some of the specific factors concerning alternative sources are noted below, and further material is to be found in [1].

(2.4.2) The EU and Norway have restrictions on future sources of energy and so R&D is required into alternative energy sources. A crucial factor is the implementation of more ecological approaches.

(2.4.3) Furthermore, the EU wants to be independent from Russia and Ukraine, so as not to be so dependent on Russian energy.

(2.4.4) Hybrid and electric vehicles are still some time away from mass availability. As GDP goes up, so too does energy consumption per capita. Notwithstanding the fact that wealthier countries use more energy, greater wealth also makes it easier to diversify energy sources.

(2.4.5) There are two main car engine types: Otto cycle and diesel. Otto cycle can use ethanol, a bio-fuel. Bio-diesel is not so straightforward to produce [1].

3 Modelling considerations

3.1 Time delays: inelastic markets

(3.1.1) Demand and supply are related to the price, although there is a 'hard bottom' to demand for oil (there is inelasticity in the demand for oil): a certain threshold amount is required regardless of the price. This is clearly a factor in transportation in general and in aviation in particular.

(3.1.2) If oil prices rise, companies seek to increase capacity and consumers seek to decrease consumption. However, there is a time delay before either market player can be effective. These two time delays (to consumer patterns and producer responses) feed into each other and the model should capture this in some way.

3.2 Modelling geopolitical shock events

(3.2.1) Although at any given time, a large geopolitical shock may be likely, that shock could come from a variety of different sources. This makes such events rather difficult to model.

3.3 Specific approaches proposed

(3.3.1) There was a general belief at the Study Group that the overall modelling solution was likely to be complicated and possibly consist of several approaches. The Study Group identified the following factors as possible components in an overall model.

(3.3.2) Adaptation of a model used to optimise the UK's energy mix on time-horizons of 2020 and 2050, subject to welfare constraints that allow the UK to meet its renewable targets. A specific model of this type is discussed in [2].

(3.3.3) Adopt a basic textbook economic approach, examining supply and demand curves.

(3.3.4) Examine models for adoption of new technologies in other industries and see how or if these could be applied to the oil industry.

(3.3.5) Consider oil prices to be a mean reverting Brownian motion process, perturbed by endogenous market player strategies and exogenous shocks.

(3.3.6) Focus on transportation, as this will be the largest demand in the future.

(3.3.7) Consider a multi-agent simulation approach, that models various sectors in the production/consumption chain as simple agents with simple behaviour.

(3.3.8) Reduce the problem by considering what proportion of a salary will one spend on oil products. Check is there a stable point of your income vs. price of oil.

(3.3.9) Check relationship between price of oil and other commodities, such as gold. This may be particularly important in times of crises, when currencies become more volatile.

(3.3.10) Inertia could be modelled as predator/prey time-delayed differential equation.

4 Development of approaches

4.1 Global welfare optimisation model

(4.1.1) We seek to optimise global welfare, in some sense. Global welfare could have various interpretations, such as reducing oil use. Ultimately, it translates to a single objective function in an optimisation routine.

(4.1.2) The starting point is a deterministic model looking at future energy supply scenarios. The global welfare goal is to compute optimal energy mixes that will meet the UK's renewable energy target in 2020 and 2050 with minimum cost. Note that it is difficult to talk about environmental targets outside a stable known region such as UK.

(4.1.3) The objective function is the cost of energy discounted to 2009 values. The original approach [2] models different parts of the market in a highly complex way. It is deterministic, and it assumes knowledge of the technology that will be available in 2050.

(4.1.4) The model consists of a deterministic optimization scenario tree, extending into the future. The various parameters on each of the branches are set in advance and probabilities are assigned to each branch. The model works in five-year blocks (but could work over any time resolution). At each node in the tree, there are three different branches e.g. high, medium and low economic growth. A further tree models technology growth as being high, medium or low.

4.2 Predator/prey models

(4.2.1) The basic equations in predator/prey models are:

$$\frac{dx}{dt} = x(\alpha - \beta y) \quad (1)$$

$$\frac{dy}{dt} = -y(\gamma - \delta x)\ . \quad (2)$$

In this equation, y is the number of predators (foxes) and x is the number of prey (rabbits). So we can consider the predators to relate to the supply of oil (in barrels) and the prey to relate to the supply of money (price in USD). In this sense, the barrels of oil are chasing after the money. This model was modified to account for time lags, as supplies of oil in the future are determined by the pricing levels today:

$$\frac{dS}{dt} = \alpha S(P_0 - P) \quad (3)$$

$$\frac{dP}{dt} = -\beta P(S_0 - S) \quad (4)$$

where P is the price, S is the oil supply and P_0 and S_0 are the historic mean levels of price and supply.

(4.2.2) These were discretised using a first order Euler approximation, i.e.:

$$P_n = P_{n-1}(1 - \beta(S_0 - S_{n-1})\Delta t) \quad (5)$$

$$S_n = S_{n-1}(1 + \alpha(P_0 - P_n)\Delta t)\ . \quad (6)$$

We are trying to solve for the parameters α and β with respect to the historic data. The approach used was to implement the above in Matlab and to compute an error term ϵ defined as $\epsilon = |S - \hat{S}|^2(P_0/S_0) + |P - \hat{P}|^2$ where P and S are the computed and $\hat{P}$ and $\hat{S}$ are the real data. Note we are multiplying by the factor P_0/S_0 to compensate for the different ranges of the data. We used 11 years' data, from 1987 to 1997 and a simple Nelder-Mead simplex optimisation algorithm (this finds local minima). The optimal values were found to be $\alpha = 2.1013$ and $\beta = 0.4843$.

(4.2.3) The resulting plot is in Figure 1. This plot that the supply vs. price follows a circular pattern. This curve has achieved neutral stability which may correspond to oil price vs. oil supply oscillations.

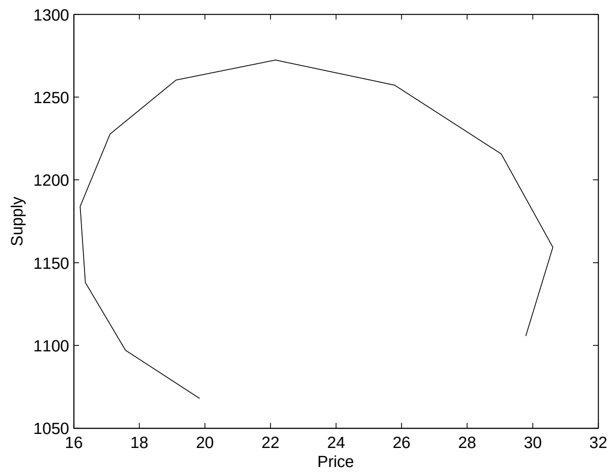

Figure 1: Price vs. supply curve achieves neutral stability when modelled with a predator/prey model.

4.3 Constrained output and technology optimisation

(4.3.1) This section adapts some ideas that have been proposed by Lanning *et al.* in connection with telecommunication services [3], which look at optimising the introduction of new technologies and management of capacity in optical communications. The analogy with the energy sector is that new technologies (such as faster optical fibre) correspond to new energy sources (such as new oil fields, renewables or biofuels). In both cases, output is constrained by the currently available sources of capacity, which are generically called 'resources' below.

(4.3.2) The key points are as follows:

(a) Demand is a function of price. There is an elastic demand D for bandwidth, with constant price elasticity ϵ (see (8) below). The elasticity introduces a nonlinearity into the problem. At least some elements in oil demand are known to be inelastic [1] and so presumably some different function would need to be used for application to oil pricing. Also, to model the possible substitution of oil by renewables, biofuels, etc, the demand may have several components, corresponding to heat, fuel, etc. Moreover, there might no longer be a single price, but instead tranches of capacity available at a sequence of increasing prices, until demand and supply are in equilibrium.

(b) In [3], the objective for optimisation is the maximisation of the net present value of cashflows from the provision of telecommunications services over some fixed time horizon. This objective function would need to be replaced with a suitable analogue for provision of heat, fuel, etc.

(c) Lanning *et al.* assume that new technologies become available at predetermined times τ_k. The cost of acquiring new technology is explicitly incorporated into the overall costs of service provision. It would be possible to consider the impact of government intervention, either to reduce costs of acquisition or operation. Interventions might also bring forward the times at which new technologies become available.

(4.3.3) An added complication occurs if we need to consider competitive provision by a number of different suppliers. Note that the Lanning *et al.* model is a monopoly. In practice, a competitive market with constraints on output can be modelled using a so-called Cournot equilibrium, in which firms choose outputs (to maximise profit) and then the market determines a price. The model has two components: the economic objectives and cost structure of individual firms (suppliers), and the equilibrium reached by the market in response to its demand curve. The following paragraphs look at each of these components in turn.

(4.3.4) **The supply side.** Firms generate profits by exploiting their resources in order to provide output in the form of a service (fuel, heat, *etc.*) to consumers. The magnitude of each firm's output is limited by the resources that it owns. If firm F_i owns resources r_i, then it may provide an output q_i in the range $[0, r_i]$ and in doing so generates an income of Pq_i, where P is the equilibrium price in the market in which the firm operates (P and q_i are determined as described below).

(4.3.5) We assume that firm F_i incurs a constant marginal cost c_i in providing each unit of service, in addition to the costs of resource access described in the next paragraph. A constant marginal cost is not a fundamental feature of the model, but it has the advantage of simplifying the explicit derivation of equilibrium market prices.

(4.3.6) Apart from the marginal cost c_i, each firm also incurs the costs of its access to resources. We assume that there is a fixed cost a_R for access to each unit of resource. Note that these costs are incurred regardless of whether or not the resources are actually used in providing a service (there is no notion here of storing resources for future use; this would be a further modification).

(4.3.7) The combined effect of the cost structure described above is that firm F_i derives a profit Π_i, given by

$$\Pi_i = (P - c_i)q_i - a_R r_i\,, \tag{7}$$

where, to summarize notation

P	is the equilibrium price of the market in which F_i operates;
q_i	is the output of F_i;
c_i	is the marginal cost to F_i of providing each unit of output;
r_i	is the resource held by F_i;
a_R	is the cost of access to resource.

Underlying this discussion is an assumption that we are constructing the profits *per unit time* of firm F_i.

(4.3.8) **The demand side.** Assume that the market reaches a Cournot-Nash equilibrium, which determines the prevailing price P at which firms provide a service to consumers. This price is reached in response to a consumer demand Q of the form

$$Q = \frac{A}{P^\epsilon}\,, \tag{8}$$

where A is the demand potential of the market and ϵ is the elasticity of demand. It is implicitly assumed that $\epsilon > 1$.

(4.3.9) As a precursor to the more general treatment that is needed here, it is useful to review the standard result that would give the equilibrium if the market contained n identical firms, with no resource costs and a sufficiently large resource that the firms' output is not constrained. The market price in the Cournot-Nash equilibrium is then given by

$$P = \frac{c}{1 - 1/(\epsilon n)}\,,$$

where c is the (common) marginal cost to each firm of providing a unit of output. This result is typically covered in graduate textbooks on microeconomics.

The consumer surplus in this situation is

$$\frac{1}{\epsilon - 1}\frac{A}{P^{\epsilon-1}}$$

and the total operator surplus is

$$\frac{1}{\epsilon n}\,\frac{A}{P^{\epsilon-1}}\,.$$

Hence the consumer surplus represents a fraction

$$\left(1+\frac{\epsilon-1}{\epsilon n}\right)^{-1} \tag{9}$$

of the combined operator and consumer surplus.

(4.3.10) The well-known results of the previous paragraph need significant refinement to deal with restrictions on the firms' individual outputs q_i, determined by the resources r_i that they hold, and the fact that firms are not identical, owing to variations in both r_i and the marginal costs c_i.

(4.3.11) To handle these effects, it is best to return to first principles. The strategy of individual firms is to choose q_i to maximize profits Π_i, as given by (7), subject to the restriction that $0 \le q_i \le r_i$. This question naturally fits into a Kuhn-Tucker framework for constrained optimization, with a solution satisfying

$$\frac{\partial \Pi_i}{\partial q_i} = \lambda_i - \mu_i\,, \tag{10}$$

where $\lambda_i, \mu_i \ge 0$ are dual variables for the constraints $q_i \le r_i$ and $q_i \ge 0$, respectively. By the standard theory for this type of problem, at the optimal solution the dual variables satisfy complementary slackness conditions

$$\lambda_i(r_i - q_i) = \mu_i q_i = 0\,.$$

In other words, λ_i is equal to zero whenever q_i is strictly less that r_i and μ_i is equal to zero whenever q_i is nonzero.

(4.3.12) Using (7) to be more explicit about the partial derivative in (10) gives

$$P - q_i\frac{\partial P}{\partial q_i} - c_i = \lambda_i - \mu_i\,.$$

Derivation of the Cournot-Nash equilibrium now involves replacing $\partial P/\partial q_i$ with dP/dQ; in other words, firms do not anticipate the effects that their choice of output might have on the output of other firms. From (8), $dQ/dP = -\epsilon Q/P$ and so (10) becomes

$$P\left(1-\frac{q_i}{\epsilon Q}\right) - c_i = \lambda_i - \mu_i\,,$$

which, in the case where $\lambda_i = \mu_i = 0$, *i.e.* $0 < q_i < r_i$, may be rewritten as

$$q_i = \frac{\epsilon A}{P^\epsilon}\left(1-\frac{c_i}{P}\right)\,, \tag{11}$$

where (8) has been used to remove Q in favour of A and P.

(4.3.13) Equation (11) gives q_i as a function of P and is one of the key steps in determining the market equilibrium. If P can be found, then this function gives the corresponding output for firm F_i, at least if the constraints $0 \leq q_i \leq r_i$ do not bind. To extend to the case when the constraints do bind, all that is needed is to cut the function off at 0 and r_i, as illustrated in Figure 2.

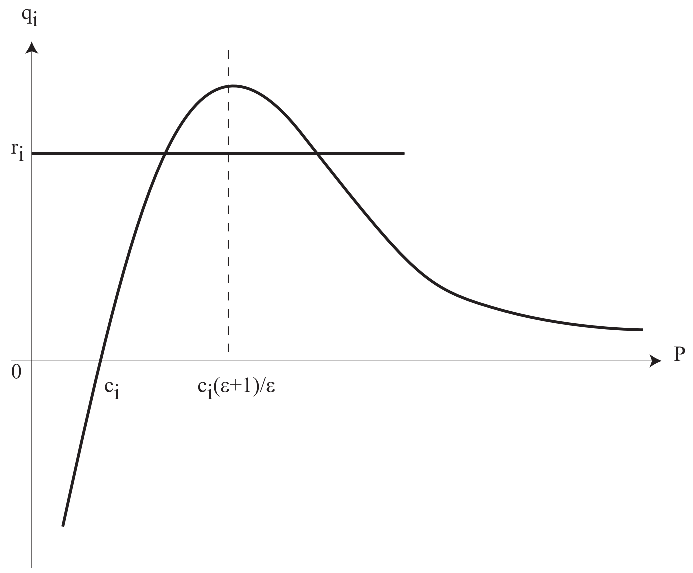

Figure 2: The relationship between the market price in equilibrium and the output of firm F_i. The output is zero if $P < c_i$ and for larger P is given by the curve shown, subject to a limiting value of r_i.

(4.3.14) Figure 2 shows the situation in which r_i is sufficiently small to affect the firm's output. From left to right, the four possible regimes are: zero output if the equilibrium price is smaller than the firm's marginal cost; a regime with low price and low output, corresponding to many firms in competition, each with a small market share; a plateau in which limited resources mean that output is constrained to the level r_i; and a regime with high price and low output, which corresponds to a monopoly position held by a single firm. If r_i is larger than the maximum value of (11) then the output will not be constrained by the limited availability of resources, no matter what the equilibrium price, and so the plateau for mid-range values of P will disappear.

(4.3.15) To calculate P, we simply sum (11) over all firms F_i in the same market and remember that

$$\sum_i q_i = Q = \frac{A}{P^\epsilon}\,. \tag{12}$$

Continuing to think of q_i as a function of P, we define

$$\phi_i(P) = \frac{q_i(P)P^\epsilon}{A}$$

and then, to satisfy (12), P is determined by requiring that

$$\sum_i \phi(P) = 1\,. \qquad (13)$$

(4.3.16) By considering the various regimes in Figure 2, we see that $\phi_i(P)$ is given explicitly by

$$\phi_i(P) = \begin{cases} 0 & \text{if } q_i = 0 \\ \epsilon(1 - c_i/P) & \text{if } 0 < q_i < r_i \\ r_i P^\epsilon / A & \text{if } q_i = r_i\,, \end{cases}$$

which in general has the form shown in Figure 3. Note that $\phi_i(P)$ is monotonically increasing from a value of 0 when $P = 0$ to an asymptote of ϵ as $P \to \infty$. Since $\epsilon > 1$ this is sufficient to guarantee that (13) has a unique solution for P.

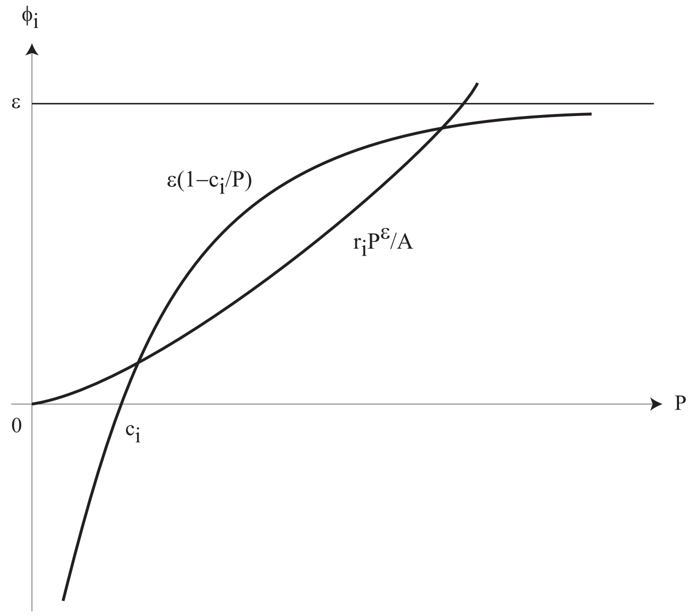

Figure 3: The quantity $\phi_i = q_i(P)P^\epsilon/A$ as a function of the equilibrium market price P; ϕ is zero if $P < c_i$ and for larger P is given by the lower of the two curves.

(4.3.17) To summarise, Lanning *et al.* discuss how to optimise the management of supply through the introduction of new sources of capacity. However, they restrict attention to a monopoly supplier. We have indicated how an output-constrained competitive market might operate, by generalising the standard Cournot-Nash equilibrium. It remains to build both of these aspects into a single model. Even then, several issues of likely importance have not been addressed, including the following.

(a) Producers might (and probably do) form a cartel and therefore avoid being in Nash equilibrium.

(b) As already noted, the relation between price and demand is expected to be more complicated than assumed above, and in particular there is evidence for a inelastic element.

(c) The existence of spot price and futures price should also be considered; futures may have significant impact on the spot price.

(d) In practice, some producers have and some do not have storage capacities (for example, oil vs. renewables).

4.4 Brownian motion with mean reversion

(4.4.1) Many commodity prices, P are modelled using a geometric Brownian motion model with mean price reversion. The mean price, M, is assumed to be the marginal cost of production of the commodity; or in the case of a cartel product such as oil, it is a pricing level at which the cartel are happy to sell the product, in the long term. The relevant equation is:

$$dP = \eta P(M - P)dt + \sigma P dz \tag{14}$$

where η is the reversion factor, σ is the asset volatility, dt is the time increment (t measured in years), dP is the price increment and dz is the Brownian term, i.e. $dz = \sqrt{dt}\mathcal{N}(0,1)$, where $\mathcal{N}(0,1)$ is a Gaussian with mean zero and unit variance.

(4.4.2) In our system we assume a fixed, long-term mean, $M_L = 40$ and a daily mean, $M(t)$. The system is driven by three separate phenomena: Brownian motion; shock events; and strategies employed by market players (consumers and suppliers).

(4.4.3) Shock events are simulated as step changes to the system. During a shock event, $M(t)$ becomes M_S, η becomes η_S and σ becomes σ_S. After the shock event, parameters η and σ revert (immediately) to typical values $\eta = 0.05$ and $\sigma = 0.3$, and $M(t)$ reverts to $M_L = 40$, the long-term mean.

(4.4.4) The influence of players in the market is characterised by assuming that both suppliers and consumers act today based on their perception of how oil price is varying over time. This perception is arrived at by observing the mean price over a short time period, τ (e.g. the previous 6 months):

$$\bar{P} = \frac{1}{\tau} \sum_{i=t-\tau}^{t} P(i)\,. \tag{15}$$

t_1	t_2	$M(t)$	η	σ
3.00	3.50	80	0.05	0.45
4.00	4.75	90	0.05	0.55
5.00	5.50	70	0.05	0.50
7.00	7.50	100	0.05	0.3500

Table 1: This table shows the series of shocks applied during Event 2. Each shock starts at time t_1, ends at time t_2, and sets $M(t)$, η and σ to the given values for the duration of the event.

(4.4.5) Both players base their decisions on the *observed mean shift*, $O_S = \bar{P} - M_L$. Initially, $M(t) = M_L$ for all t. Because of the strategies of both players,

$$M(t + \tau_2) \rightarrow M_L + f(O_S) \tag{16}$$

where function f is simply $f(O_S) = -kO_S$, for some constant k. In this way, players in the market form an opinion that oil is too expensive (or too cheap) and act to make it cheaper (or more expensive), but their actions can only become effective at some time τ_2 in the future.

(4.4.6) Two shock event scenarios were simulated. In Event 1, a single, large shock event occurred starting at $t = 5$ (start of Year 5) and ending at $t = 5.5$. The shock set $M(t) = 200$ for $5 \leq t \leq 5.5$ and set $\sigma = 0.3$ and $\eta = 0.05$ for the same period. In event 2, a series of shocks occurred, which are summarised in Table 1.

(4.4.7) Figure 4 shows how oil price behaves in the shock received in Event 1 and Figure 5 shows how the oil price behaves when it receives the shock series of Event 2.

4.5 Demand/supply curves modelled with delay differential equations

(4.5.1) Taking an economics 'textbook' approach, we can look at demand and supply curves, i.e. the price vs. quantity for demand (D) and supply (S). In simple terms, as S goes up, D goes down.

(4.5.2) Supply can be categorised as coming from either OPEC or non-OPEC countries. Demand can be categorised as coming from either the US, EU, China, Japan or India. Customers are one of either government, consumers or companies. There is a time delay in the supply and demand curve.

(4.5.3) Various factors that exert upward (↑) and/or downward (↓) pressure on the supply curve:

(a) Time lags ↑↓

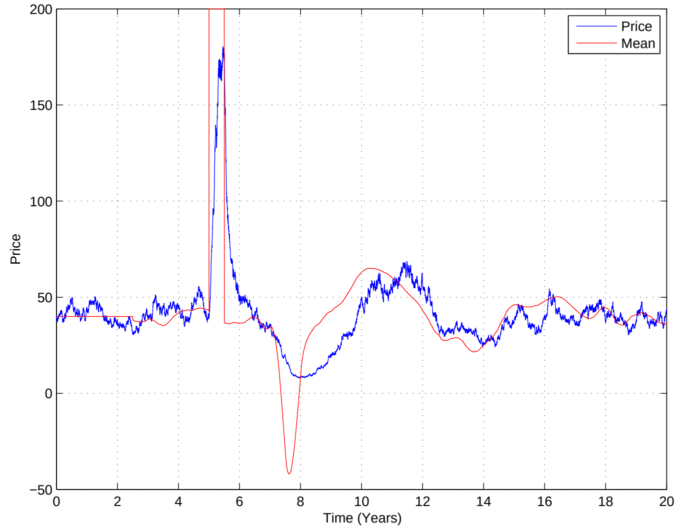

Figure 4: The blue curve shows the reaction of simulated oil price to a single large shock event (see text for details). The red curve shows the underlying mean value, $M(t)$ to which the Brownian process is trying to revert. Note the oscillations caused by the actions of the market players to perceived changes in oil price.

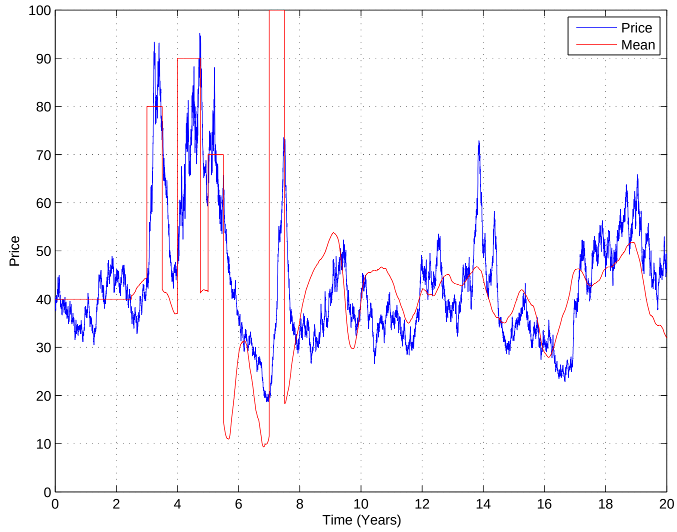

Figure 5: The blue curve shows the reaction of simulated oil price to a series of smaller shock events (see text for details). The red curve shows the underlying mean value, $M(t)$ to which the Brownian process is trying to revert. Again, note the oscillations caused by the actions of the market players to perceived changes in oil price.

(b) Investment (looking for new oil) ↑

(c) Investment in alternatives technologies ↓

(d) Ecological restrictions ↓
(e) Investment in existing infrastructure ↑

(4.5.4) Similarly, various factors exert (↑↓) pressure on the demand curve:

(a) Time lags ↑↓
(b) ‘big car’ fashion ↑
(c) Investment in alternatives technologies ↓
(d) Ecological restrictions ↓
(e) Investment in existing infrastructure ↑
(f) Increasing world population ↑
(g) Increasing world GDP ↑
(h) Politics of the EU ↓
(i) Lower prices of alternatives ↓

(4.5.5) Additionally, demand can be strongly affected by war, natural catastrophes and economic crises.

(4.5.6) We know that the oil price change depends on the supply-demand balance. If $P(t)$ is the price of energy (in USD) at time t, demand is given by $D(t)$ and supply by $S(t)$, one can set up a differential equation as follows,

$$\frac{dP(t)}{dt} = \beta(D(t) - S(t)) \tag{17}$$

where β is a sensitivity parameter.

(4.5.7) Our supply model can be written as:

$$S(t) = F(P(t - \tau)) \tag{18}$$

where F is some function (to be defined). This assumes that capacity is dependent on decisions made using the price τ years ago (e.g. $\tau \simeq 15$) so that (17) becomes a delay differential equation.

(4.5.8) A plot of F against P is given in Figure 1, p277 in [1]. A reasonable approximation to this function is

$$F = \frac{110}{3} \log\left(\frac{P}{5}\right) \tag{19}$$

(assuming $P > 5$).

(4.5.9) We experimented with several price-demand relationships that were calibrated to give supply-demand balance at around $P = 45$ (USD) and demand $D = 80$ (Million barrels of oil). Demand decreases monotonically with price and hence we propose the formulations,

$$\begin{aligned} D_A &= 80 \times 45/P \\ D_B &= \underbrace{20 \times 45/P}_{\text{Elastic}} + \underbrace{60}_{\text{Inelastic}} \\ D_C &= D_B \exp(t/80) \, . \end{aligned} \tag{20}$$

Model D_B captures aspects of the elastic and inelastic demand for oil which are not captured by D_A. Model D_C is the same as model D_B with a continuously compounded growth factor of 1/80 per annum.

(4.5.10) Combining (17, 18) and observing that (19) implies $F = F(P)$ and (20) implies $D = D(P, t)$ gives:

$$\frac{dP(t)}{dt} = \beta \left[D(P(t), t) - F(P(t - \tau)) \right] \, . \tag{21}$$

(4.5.11) We proceed in a quasi-static framework where we assume D depends on t only through P and is not intrinsically dependent on time (cases D_A and D_B but not case D_C). Then the behaviour of (21) can be examined via stability analysis and linearisation about the equilibrium point $P^* = 45$. In the standard ODE case with $\tau = 0$, the eigenvalue $\beta \left[D'(P^*) - F'(P^*) \right]$ is always negative as the rate of change of demand with respect to price is negative and the rate of change of production with respect to price is positive. Hence the system is stable.

(4.5.12) In the delay differential equation case where $\tau > 0$, we can write $P = P^* + \hat{P}$. Linearising we obtain:

$$\hat{P} = \beta \left[D'(P^*) \hat{P}(t) - F'(P^*) \hat{P}(t - \tau) \right] \, . \tag{22}$$

(4.5.13) Substituting $\hat{P}(t) = ce^{\lambda t}$ gives,

$$\lambda = \beta \left[D'(P^*) - F'(P^*) e^{-\lambda \tau} \right] \, . \tag{23}$$

We proceed via neutral stability analysis by setting $\lambda = i\omega$ (where ω is real).

(4.5.14) Equating real and imaginary parts of (23) we get,

$$0 = \beta D'(P^*) - \beta F'(P^*) \cos(\omega \tau) \tag{24}$$

$$\omega = \beta F'(P^*) \sin(\omega \tau) \, . \tag{25}$$

(4.5.15) If we re-arrange (24) and square and add to (25) squared, we obtain the following expression for ω,

$$\omega = \pm \beta \sqrt{F'(P^*)^2 - D'(P^*)^2} \tag{26}$$

which has a real solution consistent with the ansatz provided

$$|F'(P^*)| > |D'(P^*)| \,. \tag{27}$$

(4.5.16) If (27) holds, either real value of ω from (26) may be substituted into (23) via the $\lambda = i\omega$ ansatz to obtain a condition which holds between the model parameters at the loss of stability.

(4.5.17) The result is that loss of stability occurs at

$$\beta\tau = \frac{1}{\sqrt{F'(P^*)^2 - D'(P^*)^2}} PV \sin^{-1}\left(\sqrt{1 - \frac{D'(P^*)^2}{F'(P^*)^2}}\right) \,. \tag{28}$$

(4.5.18) We know that the loss of stability occurs as $\beta\tau$ increases, because we have shown stability in the ODE case $\tau = 0$.

(4.5.19) For demand profile D_A (no inelastic demand), condition (27) fails. Hence there can be no change in stability as $\beta\tau$ is varied and hence the equilibrium is stable at all parameter values.

(4.5.20) For demand profiles D_B (component of elastic demand), condition (27) holds, hence (since we fix $\tau = 15$) there is a critical price sensitivity β given by (28) above which stability is lost and limit cycle oscillations result (see Figure 6). It is curious to note that the $t \to \infty$ behaviour thus depends critically on $D'(P^*)$, which is not a quantity well calibrated from data.

(4.5.21) When we take demand function D_C, which is intrinsically time-dependent, the balance of supply and demand gives rise to an increasing equilibrium price $P^*(t)$ which is now only an equilibrium in the quasi-static sense, i.e. to perturbations which are much faster than the time-scale of intrinsic demand. However as time increases, not only does D_C, but so also does its derivative with respect to P. Hence although oscillatory behaviour may be possible at intermediate times, eventually condition (27) will fail (in a loose quasi-static sense) and the price will become monotone increasing in time (see Figure 7). Note this result is sensitive to how demand grows with time – if the proportion of elastic and inelastic demand also changed in the long-term, then the conclusion may well be different.

5 Summary

5.1 General comments

(5.1.1) A difficult problem was framed in mathematical context, leading to a lively discussion in which several approaches were proposed and discussed. Several plausible approaches were identified.

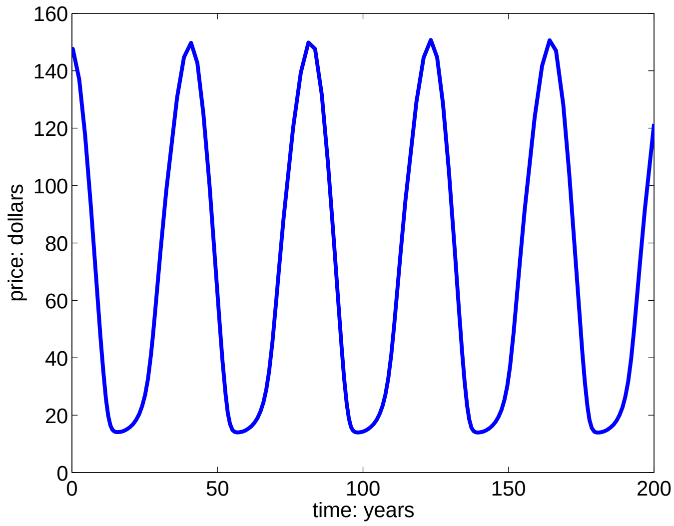

Figure 6: Limit cycle for mixed elastic/inelastic demand when demand is steady.

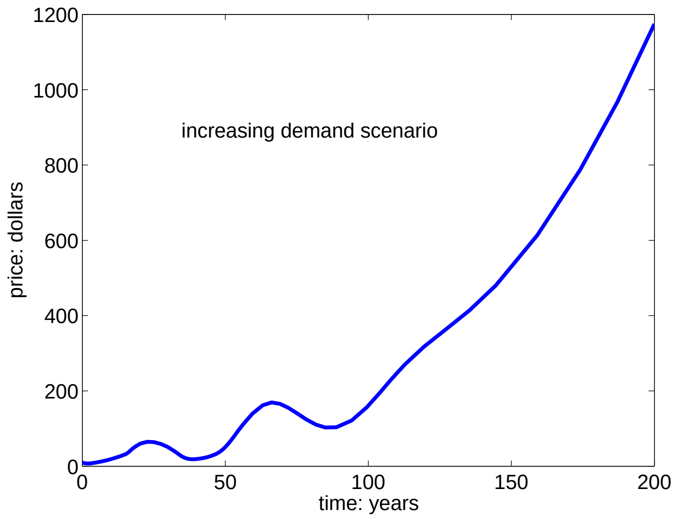

Figure 7: Limit cycle for mixed elastic/inelastic demand when demand is steadily increasing.

(5.1.2) A game-theoretic approach was proposed in Section 4.3. This approach proposed a curve relating market price in equilibrium to the output of an individual firm. In this curve, different pricing regimes could be identified. Limitations and possible extensions of this approach were also discussed.

(5.1.3) Two scenario analysis models were proposed. These incorporate time-delayed positive feedback effects caused by different players in the marketplace modifying their behaviour. The first model (Section 4.4) used a

Brownian motion approach to simulate oil as a mean reverting commodity. This model was able to realise stochastic, oscillating behaviour not unlike what is observed in reality. The second approach (Section 4.5) used a time-delayed differential equation and has parameters estimated from real data. It gave realistic price oscillations which depended on the demand scenarios.

(5.1.4) The scenario analysis work could be extended by developing either model to account in a more nuanced way for the different application areas identified in Section 1.1, in addition to dealing with the step changes associated with alternatives to oil becoming viable at different pricing levels.

Bibliography

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